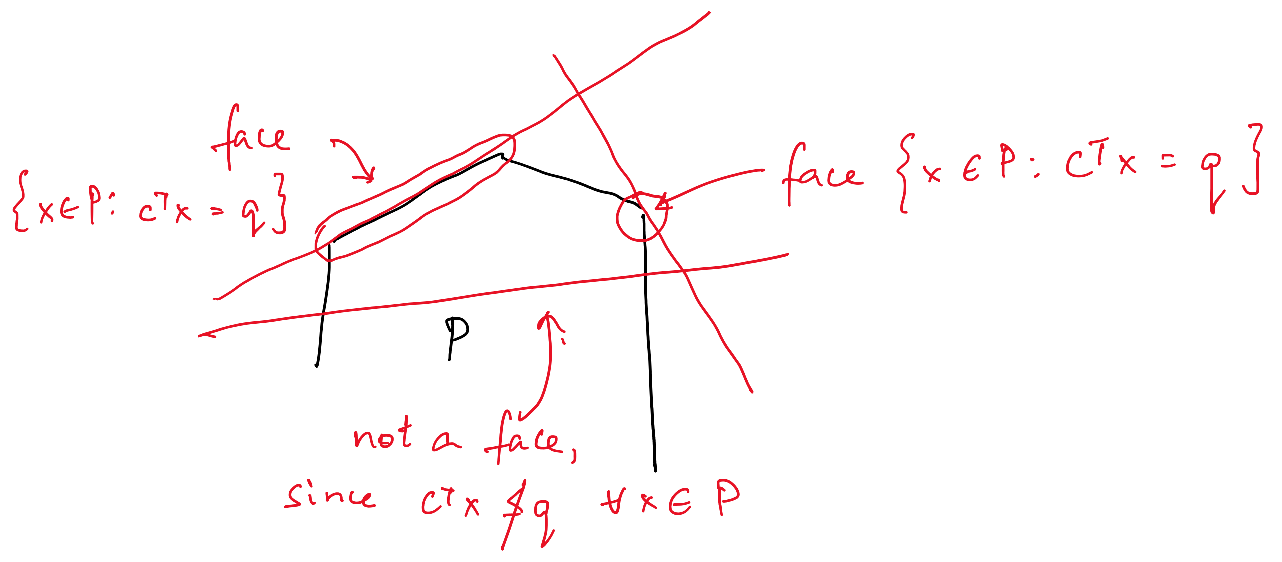
CG Refresher

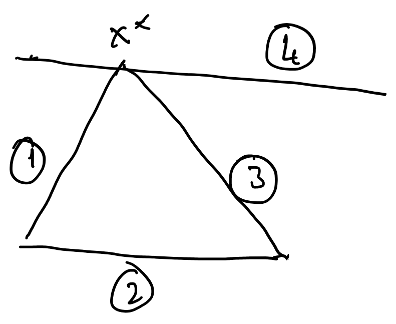
Friday, 26 August 2022 10:53 AM

- For $a \in \mathbb{R}^n$, $b \in \mathbb{R}$, the set of points $\{x \in \mathbb{R}^n : a^T x \leq b\}$ defines a halfspace of $\mathbb{R}^n$.
  The set $\{x \in \mathbb{R}^n : a^T x = b\}$ is a hyperplane
- Given $A_1, A_2, \ldots, A_m \in \mathbb{R}^n$, $b_1, \ldots, b_m \in \mathbb{R}$,
  let halfspace $H_i = \{x : A_i^T x \leq b_i\}$.
  The intersection $H_1 \cap \ldots \cap H_m$ of a finite set of halfspaces is called a polyhedron.
  Then $P = \{x : Ax \leq b\} = H_1 \cap \ldots \cap H_m$ is a polyhedron
  (where $A = \begin{bmatrix} A_1 \\ \vdots \\ A_m \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$)

  A bounded polyhedron (i.e, which is contained in a ball of finite radius) is called a polytope.
- Let $c \in \mathbb{R}^m$, $q \in \mathbb{R}$ be such that for all points x in a polyhedron P, $c^T x \leq q$.
  Then a face of P is the set $\{x \in P : c^T x = q\}$

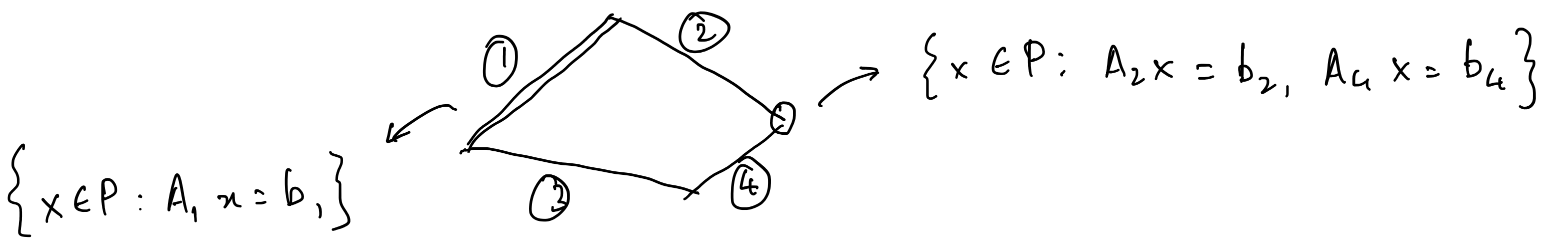


  - A vertex is a face of dimension 0
  - An edge is a face of dimension 1
  - A facet is a face of dimension $n-1$
    (assuming P is n-dimensional)
- Suppose $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ is a polytope.
  Then P is non degenerate if at any point $x \in P$, at most n equalities are tight.



Degenerate, since at x^*, ①, ③, & ④ are tight.

- Any d-dimensional face of an n-dimensional polytope is obtained by turning $n-d$ inequalities into equalities.



- A polyhedron $P \subseteq \mathbb{R}^n$ is k-dimensional if k is the largest no. s.t. $\exists k$ l.i. vectors $y^1, \ldots, y^k$ & $x^* \in P$, $\varepsilon > 0$ s.t.

$$x^* + \varepsilon y^1 \in P$$
$$x^* + \varepsilon y^2 \in P$$
$$\vdots$$
$$x^* + \varepsilon y^k \in P$$

Eg. the polyhedron $\{x \in \mathbb{R}^2 : x_1 \geq 0, x_1 \leq 0\}$ is 1-dimensional.